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WEIGHTING FUNCTION AND TRANSIENT THERMAL RESPONSE OF BUILDINGS
PART I—HOMOGENEOUS STRUCTURE

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Abstract—This paper is concerned with a theoretical investigation of transient thermal response of enclosures when the outdoor temperature follows any arbitrary function. Initially, the response of the enclosure to a unit step function is derived. The results obtained are used to define a function called the "Weighting Function", the characteristics of which depend on the thermal properties and ventilation of the enclosure considered. The response consequent on the arbitrary outdoor temperature excitation is then obtained by the use of weighting function and convolution integral. Numerical computations for a few selected cases are carried out, which demonstrate the utility of weighting functions in the study of thermal behaviour of buildings exposed to wide variations of temperature and solar radiation.

NOMENCLATURE

A, total area of the walls including roof and floor [m²];
A_e, total area of the exposed walls [m²];
A_C, A_F, A_E, A_N, etc. areas of the ceiling, floor and walls facing east and north, etc. [m²];
$A_F(t)$, temperature response of the indoor air to unit step change of outside temperature [degC];
$F(t)$, function of time defining an arbitrary outdoor temperature variation;
a, absorptivity of solar radiation;
C_a, thermal capacity of the air expressed in terms of per unit area of the exposed walls [kcal/m² degC];
C_i, thermal capacity of the internal mass expressed in terms of per unit area of the exposed walls [kcal/m² degC];
$\theta(x, t)$, temperature at position x in wall at time t [°C];
$\theta_i(t)$, inside air temperature [°C];
$\theta_{is}(t)$, temperature of internal mass [°C];
θ_{sa}, sol–air temperature [°C];
θ_0, outside air temperature [°C];
θ_C, θ_E, θ_W, etc. inside surface temperatures of ceiling and walls facing east, west, etc. [°C];
θ_{iC}, θ_{iE}, θ_{iW}, etc. contribution to the indoor air temperature by the ceiling, and walls facing east and west, etc. [°C];
$1/R_i$, inside wall surface heat-transfer coefficient [kcal/m² h degC];
$1/R_0$, outside wall surface heat-transfer coefficient [kcal/m² h degC];
$1/R_s$, heat-transfer coefficient at surface of internal mass [kcal/m² h degC];
k, thermal diffusivity [m²/h];
K, thermal conductivity [kcal/m h degC];
L, thickness of the wall [m];
x, position in wall [m];
ξ, variable of integration;
t, time [h];
I, intensity of solar radiation [kcal/m² h];
m, number of air changes per hour [1/h];
p, Laplace transform parameter;
$\phi_F(t)$, weighting function [1/h];
$\psi(p)$, function of the Laplace transform parameter, p;

$\delta(t)$, delta function;
$H(t)$, unit function;
β_n, roots of the transcendental equation occurring in the text, $n = 1, 2, \ldots$;
$\chi(\beta_n)$, function of the roots β_n;
ω, angular frequency;

$u = \dfrac{x}{K R_i}$;

$l = \dfrac{L}{K R_i}$;

$a = R_i/R_0$;

$N_s = \dfrac{K^2 R_i^2}{C_i k R_s}$;

$\tau = \dfrac{kt}{K^2 R_i^2}$;

$E = \dfrac{2R_i m C_a}{l(2 + m C_a R_i)}$;

$y_n = k \beta_n^2/K^2R_i^2, \quad n = 1, 2, \ldots$;

D_n = constants in the text, $n = 1, 2, \ldots$.

INTRODUCTION

BUILDINGS in the tropics are exposed to the influence of large fluctuations of temperature and solar radiation.* The amplitude and decay rate of transients in the heat wave transmitted through the structure are decided by the thermophysical properties of the enclosing walls. The temperature response of the indoor air to these changes depend not only on the properties of the enclosing walls but also on the internal mass and rates of ventilation of the enclosures. The behaviour of the different factors and their interplay in influencing the room response are quite complex and it is essential that they are examined to evolve an optimum design.

In this paper an attempt is made to define a function $\phi_F(t)$ to embody the complex nature of room response. The function so defined is the enclosure response to an outdoor impulsive temperature variation and it is characteristic of the particular enclosure. It includes the combined effects of ventilation, internal mass and thermophysical properties of the enclosing walls.

Since the outdoor variation in temperature is quite complex, each variation in a small interval of time can be treated as a step function change and therefore when multiplied by the corresponding $\phi_F(t)$ for that interval and summed up for all times, the required temperature response of the enclosure is obtained. Because of the very nature of the derived function, it is called the "Weighting Function". The sum of the products over all values of time can conveniently be represented by a convolution integral.

Work on the calculation of weighting functions and the consequent room response has recently been done by Fujii [1], in which the author employed an approximate technique to arrive at the weighting function. In this paper the weighting function is obtained by an exact solution of the equations governing the flow of heat in an enclosure which is influenced by ventilation and internal mass.

Recently Pratt and Ball [2] have conducted a theoretical study of transient cooling of heated enclosures when heat is produced inside the enclosure at a constant rate and a step function change occurs in the outside temperature. Since most of the equations and boundary conditions are common between the two problems, many steps in the mathematical development here have been omitted by simply giving a reference to their paper as [2]. Only those steps are retained which are essential to maintain continuity and clarity of the present analysis.

TEMPERATURE PULSE AND TRANSIENT ROOM RESPONSE

Let the temperature response of the enclosure air to a unit step function of temperature be given by $\theta_i(t) = A_F(t)$, then

$$\begin{aligned} A_F(t) &= f(p)\, H(t) \\ H(t) &= 0 \qquad \text{at } t < 0 \\ H(t) &= 1 \qquad \text{at } t > 0 \end{aligned} \tag{1}$$

as shown in Fig. 1. Here $f(p)$ is a function characterized by the room properties, where p can be identified as the Laplace-transform parameter. Obviously the Laplace-transform of $A_F(t)$ is $(1/p)\, f(p)$.

* In calculating the room response to actual outdoor climatic variation, the following formula is used for sol–air temperature which takes account of solar radiation and air temperature.

$$\theta_{sa} = \theta_0 + a I R_0$$

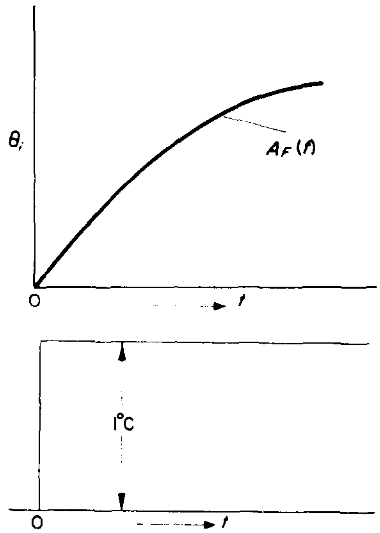

FIG. 1. Unit pulse and characteristic response curve of indoor air.

If the outside temperature takes a sudden impulsive change of the form of a delta function and if the consequent temperature of the indoor air is given by $\theta_i(t) = \phi_F(t)$, then

$$\phi_F(t) = f(p)\,\delta(t) \tag{2}$$

where $\delta(t)$ is the impulsive pulse function defined as

$$\delta(t) = 0 \qquad \text{at } t \neq 0$$
$$\delta(t) = \infty \qquad \text{at } t = 0$$
$$\int_{-\infty}^{\infty} \delta(t)\,\mathrm{d}t = 1.$$

This is shown in Fig. 2. It may also be verified that

$$\phi_F(t) = \frac{\mathrm{d}}{\mathrm{d}t} A_F(t). \tag{3}$$

Now if the outdoor temperature variation is defined by an arbitrary function $F(t)$, then the temperature of the indoor air may be computed by the convolution integral

$$\theta_i(t) = \int_0^t F(t - \xi)\,\phi_F(\xi)\,\mathrm{d}\xi \tag{4}$$

where ξ is the variable of integration.

The function $\phi_F(t)$ is called the weighting function of the enclosure for temperature variation.

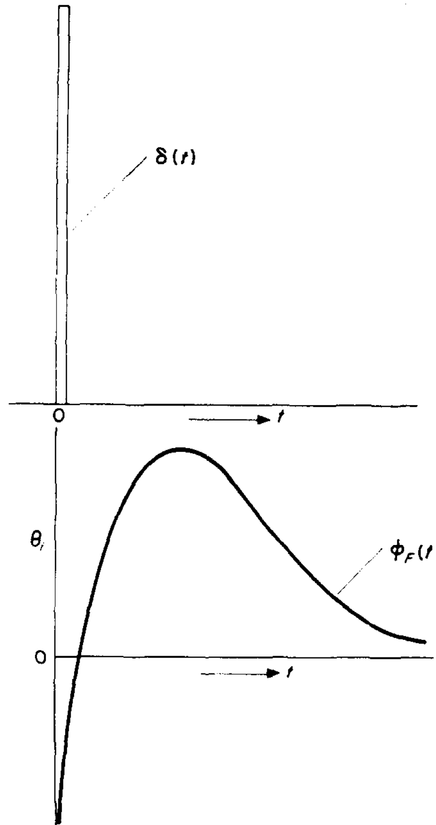

FIG. 2. Impulsive pulse and characteristic response curve of indoor air.

Consider an enclosure bounded by walls whose dimensions (length and breadth) are large compared to the thickness. The walls are assumed, for this analysis, homogeneous and isotropic. The whole enclosure is in a steady state when the temperatures inside and outside are assumed same and for convenience taken as 0°C, and as such there is no heat flow through the walls. Now suppose that the outside air temperature rises suddenly by 1 degC, the rise time being infinitesimal. The consequent transient heat flow through the enclosing wall is assumed unidirectional and perpendicular to the wall, the lateral flow being unaccounted for. The heat which arrives at the interior surfaces is disposed of in several ways. An appreciable part is absorbed by the internal mass, a part is carried away by the ventilating air, and a part establishes heat exchange between the internal surfaces and the internal mass, while the rest is absorbed by the inside air with a consequent temperature rise. Heat losses by radiative and convective processes have not been taken into account separately.

Let an enclosure, whose bounding walls are the planes $x = 0$ and $x = L$ be subjected to a temperature pulse of the form of a unit step function. It is assumed that the enclosure contains internal mass of thermal capacity C_i per unit area of the exposed walls, and is ventilated at a rate of m air changes per hour. Further, the surface heat-transfer coefficient at the exposed surfaces of the internal mass is taken as $1/R_s$, which is often taken as $1/R_i$. With these considerations, the equation of one dimensional heat transmission through the walls and the boundary conditions are

$$\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}, \quad 0 < x < L, \quad t > 0 \quad (5)$$

$$-K \frac{\partial \theta}{\partial x} = \frac{1}{R_0}(1 - \theta), \quad x = 0, \quad t > 0 \quad (6)$$

$$-K \frac{\partial \theta}{\partial x} = \frac{1}{R_i}(\theta - \theta_i), \quad x = L, \quad t > 0 \quad (7)$$

$$\theta = \theta_i = \theta_{is} = 0, \quad t > 0 \quad (8)$$

$$C_i \frac{\mathrm{d}\theta_{is}}{\mathrm{d}t} = \frac{1}{R_s}(\theta_i - \theta_{is}), \quad t > 0 \quad (9)$$

$$\frac{1}{R_i}(\theta - \theta_i) + m\, C_a (1 - \theta_i) = \frac{1}{R_s}(\theta_i - \theta_{is}), \quad x = L, \quad t > 0 \quad (10)$$

Introducing the non-dimensional variables

$$u = \frac{x}{K R_i}, \quad l = \frac{L}{K R_i}, \quad \tau = kt/K^2 R^2$$

and writing

$$N_S = \frac{K^2 R_i^2}{C_i\, k\, R_s}, \quad \alpha = R_i/R_0$$

in equations (5) to (10), and taking the Laplace-transform of the equations, we get after elimination of $\bar{\theta}$, $\bar{\theta}_{is}$ the following expression for the Laplace-transform (2) of the air temperature θ_i.

$$\bar{\theta}_i = \frac{R_s (N_s + p)\, [2\, \alpha \sqrt{(p)} + m\, C_a R_i\, \psi(p)]}{p\, \{A_1 (N_s + p)\, \psi(p) - 2\, R_s (N_s + p)\, [\alpha \sinh \sqrt{(p)}\, l + \sqrt{(p)} \cosh \sqrt{(p)}\, l] + R_i\, p\, \psi(p)\}} \quad (11)$$

where

$$\psi(p) = [\alpha + \sqrt{(p)}]\, [1 + \sqrt{(p)}]\, e^{\sqrt{(p)}\, l} - [\alpha - \sqrt{(p)}]\, [1 - \sqrt{(p)}]\, e^{-\sqrt{(p)}\, l} \quad (12)$$

$$A_1 = R_s + m\, C_a\, R_i\, R_s \quad (13)$$

and $\bar{\theta}_i$ is the Laplace-transform of θ_i defined as

$$\bar{\theta}_i = \int_0^\infty \theta_i\, e^{-p\tau}\, \mathrm{d}\tau. \quad (14)$$

To find the air temperature $\theta_i(\tau)$, we now employ the Laplace-inversion integral, the method for which has been detailed in [2]. Thus we have

$$\theta_i(\tau) = 1 - \sum_{n=1}^{\infty} \frac{R_s (N_s - \beta_n^2)\, \{\alpha\, \beta_n + m\, C_a\, R_i\, [(\alpha - \beta_n^2) \sin \beta_n\, l + \beta_n (1 + \alpha) \cos \beta_n\, l]\} \exp(-\beta_n^2\, \tau)}{\beta_n^2\, \chi(\beta_n)} \quad (15)$$

where β_n are the positive roots of the equation

$$\beta_n \tan \beta_n\, l = \frac{[m\, C_a R_i R_s (1 + \alpha) + R_i + \alpha (R_i + R_s)]\, \beta_n^4 - [R_s N_s\, m\, C_a R_i + \alpha\, R_s N_s (1 + m\, C_a R_i)]\, \beta_n^2}{(R_i + R_s + m\, C_a R_i R_s) \beta_n^4 - [\alpha\, m\, C_a R_i R_s + \alpha\, R_i + R_s N_s (1 + m\, C_a R_i)]\, \beta_n^2 + \alpha\, m\, N_s\, C_a\, R_i\, R_s} \quad (16)$$

and

$$\chi(\beta_n) = (R_i + R_s + m\,C_a\,R_i\,R_s)\,[(a - \beta_n^2)\sin\beta_n l + \beta_n(1 + a)\cos\beta_n l]$$
$$+ [(R_s + m\,C_a\,R_i\,R_s)(N_s - \beta_n^2) - R_i\beta_n^2]\left\{[1 + l/2 + al/2]\sin\beta_n l\right.$$
$$\left. + \left(\frac{l\beta_n}{2} - \frac{a\,l + a + 1}{2\beta_n}\right)\cos\beta_n l\right\} - R_s(a\sin\beta_n l + \beta_n\cos\beta_n l)$$
$$- R_s(N_s - \beta_n^2)\left[\frac{l}{2}\sin\beta_n l - \left(\frac{a\,l + 1}{2\beta_n}\right)\cos\beta_n l\right] \tag{17}$$

RESPONSE DUE TO AN IMPULSIVE PULSE

From equations (3) and (15) the temperature response due to an impulsive pulse or the weighting function $\phi_F(t)$ is

$$\phi_F(t) = \sum_{n=1}^{\infty} \frac{B_n}{\chi(\beta_n)} \exp(-\beta_n^2\,\tau) \tag{18}$$

where

$$B_n = \frac{k}{K^2 R_i^2} R_s(N_s - \beta_n^2)\,\{a\,\beta_n + m\,C_a\,R_i\,[(a - \beta_n^2)\sin\beta_n l + \beta_n(1 + a)\cos\beta_n l]\} \tag{19}$$

and β_n, $\chi(\beta_n)$ are as given in equations (16), (17).

For numerical computation of weighting functions for different values of the parameters, the transcendental equation is solved and the values of β_1, β_2, etc., obtained. It was found that except for small values of time, the roots higher than the second do not significantly contribute to the computed results. At time $t = 0$, the results are likely to be in error, but the series (18) may be approximated as shown below. Assuming that $\beta_3 \gg \beta_2$ and $\beta_4 \gg \beta_3$, etc., we take the limit of terms following $n = 2$ for large β_n in (18) and get

$$\phi_F(t) = \frac{B_1}{\chi(\beta_1)} \exp(-k\,\beta_1^2\,t/k^2\,R_i^2) + \frac{B_2}{\chi(\beta_2)} \exp(-k\,\beta_2^2\,t/k^2\,R_i^2) - \frac{2\,k\,R_i\,m\,C_a \sum_{n=3}^{\infty} \beta_n \tan\beta_n l \exp(-k\,\beta_n^2\,t/K^2\,R_i)}{l\,K^2\,R_i^2\,(2 + m\,C_a\,R_i)}. \tag{20}$$

From (20) it is seen that for $t > 0$ the contribution of the terms under the summation is very insignificant but at $t = 0$ it is very large since the series $-\sum_{n=3}^{\infty} \beta_n \tan\beta_n l$ is divergent. The summation is therefore approximated by the Delta Function as

$$\phi_F(t) = \frac{B_1}{\chi(\beta_1)} \exp(-k\,\beta_1^2\,t/K^2\,R_i^2) + \frac{B_2}{\chi(\beta_2)} \exp(-k\,\beta_2^2\,t/K^2\,R_i^2) + E_1\,\delta(t) \tag{21}$$

where

$$E_1 = \frac{2\,k\,m\,C_a}{l\,K^2\,R_i\,(2 + m\,C_a\,R_i)}.$$

It is interesting to note that the coefficient of $\delta(t)$ depends on the ventilation and thermal capacity of the air, and also on the physical properties of the wall material. This may be identified as the instantaneous heat which enters the enclosure following an impulsive change of the outdoor air temperature. The weighting function may be written in its final form as

$$\phi_F(t) = \frac{k}{K^2 R_i^2} [D_1 \exp(-y_1 t) + D_2 \exp(-y_2 t) + E\, \delta(t)] \tag{22}$$

where

$$D_n = \frac{R_s (N_s - \beta_n^2) \{a \beta_n + m C_a R_i [(a - \beta_n^2) \sin \beta_n l + \beta_n (1 + a) \cos \beta_n l]\}}{\chi(\beta^u)}$$

$$n = 1, 2, \ldots \tag{23}$$

$$y_1 = \frac{k \beta_1^2}{K^2 R_i^2}, \quad y_2 = \frac{k \beta_2^2}{K^2 R_i^2} \tag{24}$$

$$E = \frac{2 R_i m C_a}{l (2 + m C_a R_i)}. \tag{25}$$

Weighting functions of a few enclosures having typical wall constructions are shown in Figs. 3–7.

CHARACTERISTIC WEIGHTING FUNCTION OF ENCLOSURES

The theoretical results derived are applied to the computation of weighting functions of two types of structures, one having a massive construction and the other a light construction. In order to study the influence of enclosure-size on weighting functions, a small room and a large house are considered. The small room may be like one of those in the Institute's Field Project built up especially for studies on the thermal response of rooms of various constructions. It may also be an apartment inside a building. The large house visualized may be a complete building containing many small rooms.

Weighting function is influenced by internal mass and the rates of ventilation. Their influence is studied in a generalized manner as given in Tables 1 and 2. Because of its definition, the value of C_a is dependent on the ratio, vol/exposed wall area of the enclosure. For instance in code I, Table 1, the value of $C_a = 0{\cdot}5 \times 0{\cdot}3 = 0{\cdot}15$ (thermal capacity of air $= 0{\cdot}3$ kcal/m^3 degC).

Unexposed internal walls, furnishing and commodities form the internal mass. As no exact value can be prescribed for the internal mass due to furnishing, etc., only arbitrary values are given in Tables 1 and 2. Nevertheless, for internal walls shared by neighbouring rooms only half the thickness is considered as contributory to the internal mass. In a large house, the full thickness of interior walls is considered as effective internal mass.

Heat flow through floors particularly those on raised plinths is quite complex. Experimental observation in the Test Houses indicate that they maintain temperatures close to those of indoor air. The floor has therefore been left out of consideration.

Computation of the roots and constants have been carried out [Table 3] for the very generalized cases in Tables 1 and 2. Only the first two roots are included as the contribution of the higher order roots is insignificant. The computed weighting functions are plotted for different hours in Figs. 3–7.

Transient response of various types of enclosures to impulsive variations in outdoor air-temperature are manifest in their characteristic weighting function. Figure 3 shows the influence of ventilation and internal mass on the characteristic weighting functions of a small room enclosed by heavy masonry walls. In general, the room responds very slowly to a sharp variation in outside temperature. When the room is empty, the rate of temperature rise of the indoor air attains a peak, although small, after about four hours of the outside variation. Ventilation of the room causes the peak to occur earlier and also enhances the subsequent cooling of the room. Addition of internal mass suppresses the peak considerably but prolongs the rate of cooling. Influence of ventilation on rooms with internal mass is more pronounced than without it. Further increase in internal mass seems to alter the pattern of functions. This may be ascribed to errors involved in computation

Table 1. Heavy structure. Table of data used for computation of weighting functions

Brick Enclosure	Enclosure Type	Code No.	Vol/Exposed wall area	C_a kcal/m² °C	m 1/hr	C_i kcal/m² °C
$L = 20$ cm	Small Room	1	0·5	0·15	1	0, 20
$K = 0{\cdot}87$ kcal/m hr °C		2	0·5	0·15	6	0, 20, 68
$k = 2{\cdot}4 \times 10^{-3}$ m²/hr		3b	1·0	0·3	1	20
$R_i = R_s = 1/6$ m²hr°C/kcal		4c	1·0	0·3	6	68
$R_0 = 1/20$ m²hr°C/kcal	Large House	5c	1·0	0·3	20	68
Thermal capacity of air = 0·3 kcal/m³ °C		6	1·5	0·45	1	0
Specific heat of brick = 0·2 kcal/kg °C		7	1·5	0·45	6	20
Density = 1770 kg/m³		8	2·0	0·6	1	68
		9	2·0	0·6	20	68

Notes: a, b, c denote respectively, zero, moderate and high internal mass.
Codes 1 and 2 refer to a small room all the six faces of which are exposed (upper storey). The internal mass postulated is in the form of furnishing.
Codes 3b, 4c and 5c refer to a small room, whose three faces are exposed. The value of effective internal mass is decided by unexposed faces and furnishing. Codes 6, 7 refer to a large house whose all faces are exposed (upper storey), the internal mass being in the form of furnishing. Codes 8, 9 refer to a large house of which five faces are exposed. Interior walls and furnishing are considered as internal mass.

Table 2. Light weight structure. Table of data used for computation of weighting functions

Enclosure of wood	Enclosure type	Code No.	Vol/Exposed wall area	C_a kcal/m² °C	m 1/hr	C_i kcal/m² °C
$L = 2{\cdot}5$ cm	Room Small	11a	0·5	0·15	1	0
$K = 0{\cdot}11$ kcal/m hr °C		11b	0·5	0·15	1	3
$k = 1{\cdot}1 \times 10^{-3}$ m²/hr		12a	0·5	0·15	6	0
$R_i = R_s = 1/6$ m²hr °C/kcal		13b	1·0	0·3	1	3
$R_0 = 1/20$ m²hr °C/kcal		14b	1·0	0·3	6	3
Specific heat of wood = 0·4 kcal/kg °C	Large House	15a	1·5	0·45	1	0
		16	1·5	0·45	6	0, 10
Density = 250 kg/m³		17c	2·0	0·6	1	10
		18c	2·0	0·6	20	10

Notes: a, b, c denote respectively, zero, moderate and high internal mass.
Codes 11a, 11b, 12a refer to a small room, all the six faces of which are exposed (upper storey). The internal mass postulated is in the form of furnishing.
Codes 13b, 14b refer to a small room, whose three faces are exposed. The value of effective internal mass is decided by unexposed faces and furnishing.
Codes 15a and 16 refer to a large house whose all faces are exposed (upper storey). The internal mass is in the form of furnishing.
Codes 8 and 9 refer to a large house of which four faces are exposed. Interior walls and furnishing are considered as internal mass.

Table 3. Values of the computed coefficients and roots for the different cases considered under Tables 1 and 2

Reference to Code No. in Tables 1 & 2	Heavy structure (l = 1·38) Root β_1	Root β_2	Coefficient D_1	Coefficient D_2	Reference to Code No. in Tables 1 & 2	Light structure (l = 1·38) Root β_1	Root β_2	Coefficient D_1	Coefficient D_2
1(a)	0·96	2·85	1·065	−2·800	11(a)	0·97	2·95	+0·940	−2·313
1(b)	0·79	1·43	0·578	−0·233	11(b)	0·42	1·20	+0·142	+0·378
2(a)	1·025	2·95	1·038	−2·146	12(a)	1·025	2·96	+1·038	−2·146
2(b)	0·81	1·43	0·593	−0·220	13(b)	0·428	1·23	+0·148	+0·387
2(c)	0·54	1·25	0·176	0·293	14(b)	0·48	1·25	+0·168	+0·39
3(b)	0·735	1·42	0·068	−0·059	15(a)	1·02	2·92	+1·009	−2·07
4(c)	0·555	1·25	0·212	0·310	16(a)	1·119	2·97	+0·886	−1·568
5(c)	0·655	1·28	0·213	+0·276	16(c)	0·48	1·23	−0·020	+0·430
6(a)	1·02	2·95	1·009	−2·074	17(c)	0·49	1·21	−0·073	+0·454
7(b)	0·942	1·44	0·617	−0·189	18(c)	0·44	1·25	−0·10	+0·349
8(c)	0·51	1·24	0·163	+0·304					
9(c)	0·744	1·28	0·213	+0·276					

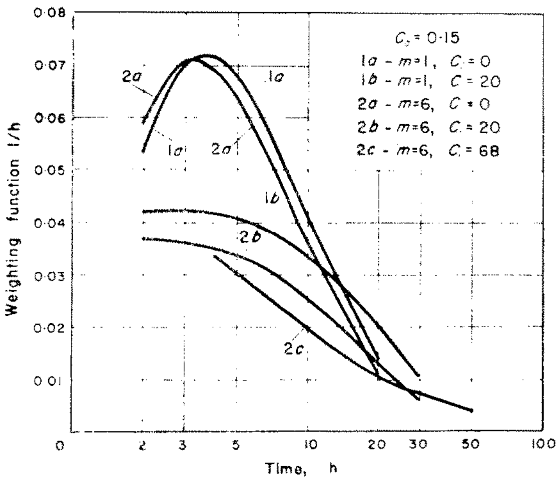

FIG. 3. Weighting functions of a small room of heavy weight construction, C_a = 0·15.

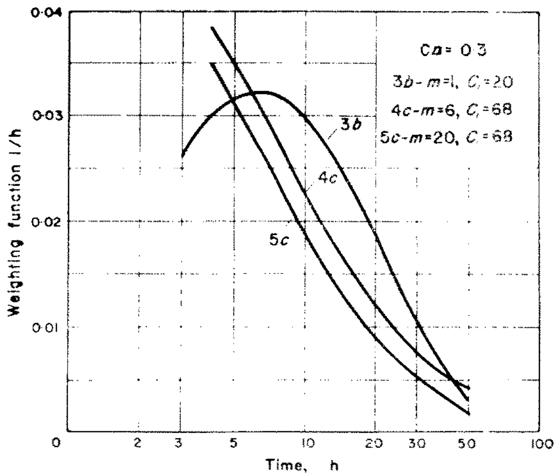

FIG. 4. Weighting functions of a small room of heavy weight construction, C_a = 0·3.

of the functions during small hours when the roots greater than the second play significant role. Figure 4 exhibits the characteristics when the room contains three unexposed walls and internal mass. The response is delayed and the rate of temperature rise is lowered [cf. 1(*b*) and 3(*b*)]. This shows that reduced number of exposed walls of a room will lower temperature rise. Additional furnishing causes further lowering of the rate of temperature rise.

The pattern of the characteristic functions (Fig. 5) for a house having large ratio of cubic volume and wall area is more or less the same as in a small room. When the houses have only air infiltration ($m = 1$) the rate of temperature rise in a small house is slightly greater [1(*a*), 6(*a*)]. With increase in ventilation ($m = 6$), the rate of rise in a large house is greater initially, but finally it is lower than in a small house [2(*b*), 7(*b*)]. Thus the ventilation of a larger house enhances the cooling rate.

Response of a light weight structure to a

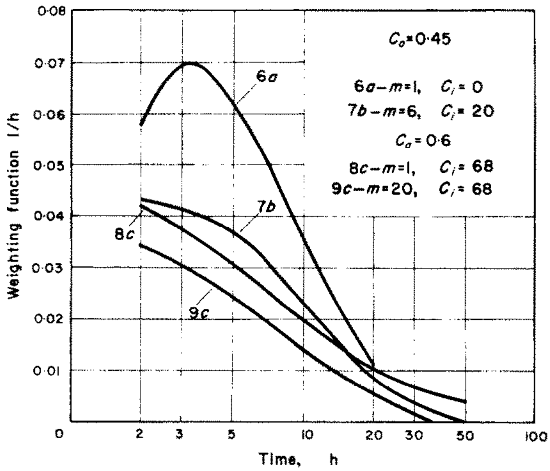

FIG. 5. Weighting functions of a large house of heavy weight construction, $C_a = 0{\cdot}45$.

sudden variation in outdoor temperature is shown in Fig. 6 and 7. An empty enclosure with wooden walls transmits heat indoors almost instantaneously and the inside temperature changes fast. The peak rate of temperature rise is about thirty times compared to a similar enclosure having brick walls [cf. 1(*a*), 11(*a*), and 6(*a*), 15(*a*)]. The influence of ventilation in small and large houses is similar to that in the heavy structure. Addition of internal mass reduces the rate of temperature rise by absorbing the heat that penetrates the thin walls but its performance does not approach that of a masonry structure.

RESPONSE TO PERIODIC VARIATION OF AIR TEMPERATURE

Weighting functions thus computed can be used to determine the response of an enclosure to any periodic change in outdoor air temperature with the help of the convolution integral. If the temperature variation of the outdoor air follows a function $F(t)$, then the corresponding enclosure response is given by equation (4). In particular for periodic variation given by

$$F(t) = F_0 + F_1 \cos(\omega t - \epsilon) \qquad (26)$$

where F_0 and F_1 are constants, and ω the frequency, the response of the enclosure is

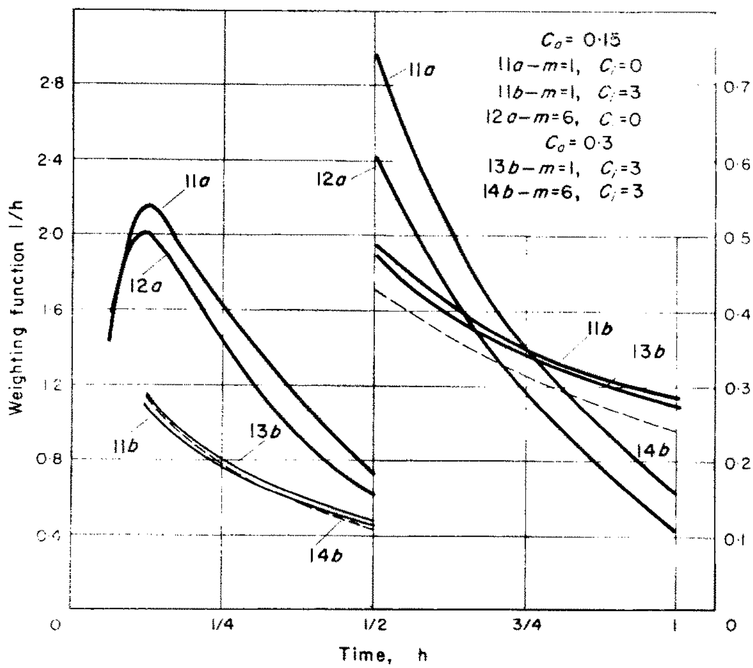

FIG. 6. Weighting functions of a small room of light weight construction, $C_a = 0{\cdot}15$.

$$\theta_i(t) = \frac{k}{K^2 R_i^2} \left\{ \left[\frac{D_1}{y_1} [1 - \exp(-y_1 t)] + \frac{D_2}{y_2} [1 - \exp(-y_2 t)] + E \right] F_0 + \left(\frac{D_1 y_1}{y_1^2 + \omega^2} + \frac{D_2 y_2}{y_2^2 + \omega^2} \right) F_1 \cos(\omega t - \epsilon) + \left(\frac{D_1}{y_1^2 + \omega^2} + \frac{D_2}{y_2^2 + \omega^2} \right) F_1 \, \omega \sin(\omega t - \epsilon) + \left(\frac{\omega}{y_1^2} \sin\epsilon - \frac{1}{y_1} \cos\epsilon \right) \exp(-y_1 t) + \left(\frac{\omega}{y_2^2} \sin\epsilon - \frac{1}{y_2} \cos\epsilon \right) \exp(-y_2 t) \right\}. \tag{27}$$

If the enclosure is subjected to this periodic variation for a long time, then the effects of transients vanish, and we have

$$\theta_i(t) = \frac{k}{K^2 R_i^2} \left[\left(\frac{D_1}{y_1} + \frac{D_2}{y_2} + E \right) F_0 + \left(\frac{D_1 y_1}{y_1^2 + \omega^2} + \frac{D_2 y_2}{y_2^2 + \omega^2} \right) F_1 \cos(\omega t - \epsilon) + \left(\frac{D_1}{y_1^2 + \omega^2} + \frac{D_2}{y_2^2 + \omega^2} \right) \omega F_1 \sin(\omega t - \epsilon) \right]. \tag{28}$$

RESPONSE TO SOLAR RADIATION

Weighting functions obtained above are for an enclosure subjected to the influence of variations in air temperature only. The response consequent on variations in the incident solar radiation can be computed through sol–air temperature and the same weighting functions provided the enclosure has restricted or no ventilation. This is because the step function in the equilibrium equation (10) represents a sudden change in the outdoor air temperature. If the concept of sol–air temperature is extended to represent this function, the heat transfer by ventilation in equation (10) would be exceedingly high unless the rate of ventilation m is small or the difference between sol–air temperature and air temperature is negligible. It may be observed from equations (16), (17) and (19) that the contribution on weighting function of the ventilation term for small values of m is small compared to other terms.

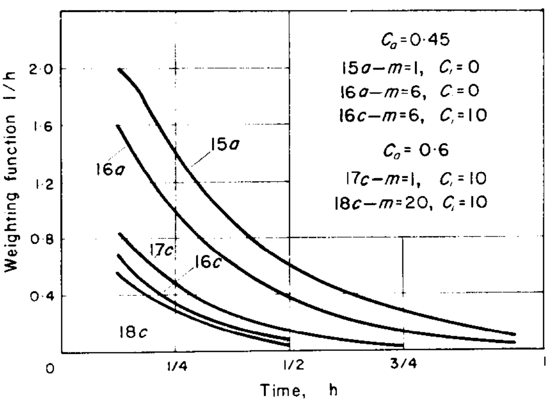

FIG. 7. Weighting functions of a large house of light weight construction, $C_a = 0{\cdot}45$ and $0{\cdot}6$.

ROOM RESPONSE TO PERIODIC VARIATION OF OUTDOOR TEMPERATURE AND SOLAR RADIATION

It will be apparent from the discussion and derivation of the weighting functions that the final room response given by equation (28) due to a periodic variation of outside air temperature holds only for a homogeneous room. All the exposed walls of this room are composed of the same material and have the same excitation. But in actual practice, walls of the room may be of different material composition, for instance the roof may be of cement concrete and walls of brick. The temperature excitations for the, exposed walls are also different because of different intensities of solar radiation incident on them. The room response of composite construction and having different exposures can be deduced on the basis of the equilibrium conditions. The amount of heat transfer from the interior surfaces to the indoor air may be obtained by multiplying the difference of surface and air temperatures by their respective wall surface heat-transfer coefficients. The usual process is to take an over-all surface heat-transfer coefficient or surface conductance which takes into account the convective as well as the radiative heat transfer. The conductance values for horizontal and vertical surfaces are different and also depend on temperatures of the surfaces. Nevertheless, to simplify calculations, surface conductances are taken as constant and same for all surfaces. The simplified equilibrium equation is thus given by

$$\frac{1}{R_i}\left[A_N\,\theta_N + A_S\,\theta_S + A_W\,\theta_W + A_E\,\theta_E + A_C\,\theta_C + A_F\,\theta_F - A\,\theta_i\right] + m\,A_e\,C_a\,(\theta_0 - \theta_i) = 0 \tag{29}$$

where θ_C, θ_E, etc., are the inside surface temperatures of the ceiling, east wall, etc., θ_0 is the outdoor air temperature, A's are the respective areas and A_e is the total exposed area.

In order to calculate the inside air temperature of a composite enclosure from equation (29) it is necessary to determine the inside surface temperatures of the exposed walls. The contribution of each surface to the indoor air is considered separately. Treating the unexposed walls as internal mass, the temperatures of the inside surfaces are given by the following equations

$$\begin{aligned}\theta_C &= (1 + m\,C_a\,R_i)\,\theta_{iC} - m\,C_a\,R_i\,\theta_0\\ \theta_E &= (1 + m\,C_a\,R_i)\,\theta_{iE} - m\,C_a\,R_i\,\theta_0\\ &\ldots\end{aligned} \tag{30}$$

where θ_{iC}, θ_{iE}, etc., are the contributions to the indoor air temperature by the ceiling and wall facing east, etc. These are determined from their respective weighting functions and sol–air temperatures.

Response of an enclosure subjected to changes

Table 4. Field test room specifications and sol–air temperature

Surface	Composition	Roots and coefficients of weighting functions				Sol–air temperature °C for the surface
		β_1	β_2	D_1	D_2	
Horizontal roof 3·46 × 2·88 m	17·8 cm thick cement concrete $K = 1{\cdot}54$ kcal/m²hr°C $k = 0{\cdot}0031$ m²/hr $1/R_i = 7{\cdot}32$ kcal/m²hr°C $1/R_0 = 17{\cdot}08$ kcal/m²hr°C $l = 0{\cdot}85$	1·15	2·58	1·957	−3·343	36·97–18·3 cos (0·262t–0·265)
East wall 3·46 × 3·22 m	25·4 cm thick brick $K = 0{\cdot}564$ kcal/m²hr°C $k = 0{\cdot}0015$ m²/hr $1/R_i$, $1/R_0$, same as for the roof $l = 3{\cdot}3$	0·39	1·082	0·200	−0·398	34·2–11·35 cos (0·262t–0·19)
South wall 3·22 × 2·88 m	ditto					32·08–10·2 cos (0·262t–0·57)
West wall 3·48 × 3·22 m	ditto					33·67–11·5 cos (0·262t–0·82)
North wall 3·22 × 2·88 m	ditto	Unexposed	—	—	—	—
Floor 3·46 × 2·88 m	Cement floor on 50 cm high plinth	—	—	—	—	—

Notes: North wall of the test room is unexposed and shared by the anteroom, half of its thickness is taken as internal mass, $C_i = 8$ kcal/m² °C. Heat flow through the floor not taken into account.
The enclosure considered is unventilated ($m = 0$).

of air temperature and solar radiation may be computed as follows:

(*a*) When the enclosure is unventilated, sol–air temperature will represent the function $F(t)$ in (26). The solution so obtained will be exact.

(*b*) When the enclosure has restricted ventilation, that is when m is small, the function $F(t)$ may again be considered as the sol–air temperature. This will introduce error, although not of a significant magnitude, in the ultimate response. This condition is realized during the periods of intense summer sun, when the houses in Northern India are usually shut out to preserve coolth.

(*c*) For an enclosure having high rates of ventilation approximate results may be obtained by first calculating the contribution θ_{iC}, etc., of each wall to the room air temperature under the conditions mentioned in (*a*). The results so obtained are then used to find the inside surface temperatures θ_C, etc., through the formulae (30) with $m \neq 0$. Surface temperatures are then substituted in equation (29) to determine the inside room temperature θ_i.

Response of an unventilated room ($m = 0$) subjected to intense solar radiation and air temperature (in shade) as availing during summer in North India has been evaluated. This is one of the test rooms constructed in the field for studies on thermal response and indoor climate of buildings. The rooms are erected 12 metres apart to have unobstructed temperature, solar and air exposure. Entrance to each test room is through a small door on the north partition wall and a 1·2 m wide anteroom which serves to check air infiltration. Round the clock observations on the indoor temperature response of the room subjected to the sol–air temperatures in Table 4 were taken on two typical summer days by means of remote reading thermocouples.

The temperature response of the indoor air computed by the method of weighting functions is given in Fig. 8. The predicted temperatures agree, in general, with the measured values. Nevertheless, there is a prominent divergence during the early hours of the experiment, viz.

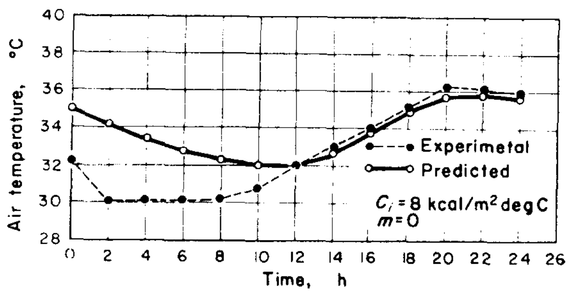

FIG. 8. Thermal response of Field Test House—Computed and experimental curves.

12 midnight to 10 a.m. The disagreement gradually reduces with the progress of the experiment and vanishes after about 10 a.m. Several factors contribute to this difference between the experimental and the predicted values. The primary cause is attributed to the inside thermal condition of the room which is much different from the condition postulated in the theory at time $t = 0$. The theory requires that the room is subjected to the same sol–air temperature long before the commencement of the experiment. But actually the room was having some ventilation and was not exposed to the same temperature cycle as in Table 4, prior to the start of the experiment. Secondly, the heat flow through the floor which is not insignificant has not been taken in calculations. Lastly, the radiative and convective exchange inside the room have been considered only partly through the overall inside surface heat-transfer coefficient $1/R_i$ and some error may also be caused by this approximation.

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Résumé—Cet article a pour objet une étude théorique de la réponse thermique transitoire d'enceintes quand la température extérieure suit une loi arbitraire quelconque. Initialement, la réponse de l'enceinte à une fonction échelon-unité est obtenue. Les résultats obtenus sont utilisés pour définir une fonction appelée la "fonctio de pondération", dont les caractéristiques dépendent des propriétés thermiques et de la ventilation de l'enceinte considérée. La réponse à l'excitation de température extérieure arbitraire est alors obtenue à l'aide de la fonction de pondération et d'une intégrale de convolution. Des calculs numériques pour quelques cas choisis ont été conduits, qui dépontrent l'utilité des fonctions de pondération dans l'étude du comportement thermique de bâtiments exposés à de grandes variations de température et d'ensoleillement.

Zusammenfassung—In einer theoretischen Untersuchung wird das instationäre thermische Verhalten von Räumen behandelt für Änderungen der Aussentemperatur nach einer beliebigen Funktion. Zuerst wird das Verhalten der Räume für eine Einheitsschrittfunktion abgeleitet. Mit diesen Ergebnissen lässt sich eine Funktion definieren, die "Gewichtsfunktion" genannt wird. Sie ist durch die thermischen Eigenschaften und die Belüftung des betrachteten Raumes charakterisiert. Das Verhalten entsprechend der willkürlichen Aussentemperatur wird mit Hilfe der Gewichtsfunktion und eines Faltintegrals ermittelt. Numerische Rechnungen sind für einige ausgewählte Fälle durchgeführt. Sie zeigen die Nützlichkeit der Gewichtsfunktionen für die Untersuchung des thermischen Verhaltens von Gebäuden, die einem grossen Variationsbereich der Temperatur und der Sonnenstrahling ausgesetzt sind.

Аннотация—Данная статья посвящена теоретическим исследованиям нестационарной тепловой реакции ограждений, когда температура окружающей среды описывается по произвольному закону. Вначале определяется реакция ограждения в виде единичной функции. Полученные результаты используются для определения функции, называемой «весовой функцией», характеристики которой зависят от теплофизических свойств и вентиляции рассматриваемого ограждения. Тогда реакция на произвольное изменение температуры окружающей среды определяется путём использования весовой функции и интеграла со сверткой. Выполнены численные расчёты для нескольких случаев, демонстрирующие применимость весовых функций при изучении теплового поведения сооружений, подверженных действию солнечной радиации и температур, изменяющихся в широком интервале.